Develop Solving Problems with Unit Rates for Ratios with Two Fractions

Purpose

- **Develop** strategies for comparing unit rates that are calculated from fractions.
- **Recognize** that ratios with fractions can be simplified by solving division problems.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A: 12 : 1	B: $\frac{24}{2}$
C: $\frac{1}{4} \div 3$	D: $3 \div \frac{1}{4}$

Possible Solutions

B and C both equal 12, while D equals $\frac{1}{12}$.

A is a ratio.

B is a fraction.

C and D are expressions including division.

WHY? Support students' ability to understand, evaluate, and express ratios in different formats.

DEVELOP ACADEMIC LANGUAGE

WHY? Support students as they listen for understanding.

HOW? As students participate in partner and group discussions, have them listen actively by using engaged body language such as facing the speaker and nodding to show understanding. Remind them to try to understand each other's strategies. Provide a sentence frame to support students as they ask for clarification: *I think you said _____. Is that correct?*

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Make sure students understand that important information is given in the illustration of a mobile phone. If no one notices the different quantities, ask: *What do you notice about the quantities for Kyle and Julio?*

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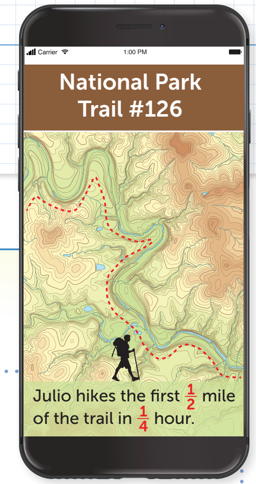

Read and try to solve the problem below.

Julio and Kyle hike along the same trail. Kyle hikes the first 6 miles in 2 hours. Who hikes more quickly?

TRY IT

Math Toolkit double number lines, fraction strips, fraction tiles, grid paper

Possible work:

SAMPLE A

Julio's Speed				
Miles	$\frac{1}{2}$	$\frac{2}{2} = 1$	$\frac{3}{2}$	$\frac{4}{2} = 2$
Hours	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4} = 1$

Kyle's Speed		
Miles	6	3
Hours	2	1

Kyle hikes more quickly.

SAMPLE B

Kyle's ratio of miles to hours is 6 : 2.

$6 \div 2 = 3$

Julio's ratio of miles to hours is $\frac{1}{2} : \frac{1}{4}$.

$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \cdot 4$

$= 2$

Kyle hikes more quickly.

DISCUSS IT

Ask: Why did you choose that strategy to find who is hiking more quickly?

Share: I knew . . . so I . . .

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. As they work, remind students to listen actively to their partner and to show understanding by nodding. If students need support in getting started, prompt them to ask each other questions such as:

- *How did you begin to think about this problem?*
- *How does your strategy show who hikes more quickly?*

Common Misconception Listen for students who try to compare the quantities by using subtraction, such as $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$. As students share their strategies, have them identify the different units for the quantities of the problem, which are miles and hours. Ask: *Can $\frac{1}{4}$ hour be subtracted from $\frac{1}{2}$ mile? Why or why not?* Listen for students to explain to one another that it cannot because the quantities represent different kinds of measurement. Instead, they should be compared with a ratio.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- tables or double number lines that compare the distance for each hiker at various times
- (misconception) equations that utilize subtraction rather than multiplication or division
- equations that calculate the rate, or speed in miles per hour, for each hiker

Facilitate Whole Class Discussion

Use turn and talk to have students share, process, and refine their strategies for solving the problem. Then call on selected students to share their strategies with the class. Remind students to look at those speaking and to try to understand their ideas.

Guide students to **Compare and Connect** the representations. Have students evaluate each representation and decide if it represents the ratio of distance to time for each hiker.

ASK *How do different solutions show the distance that each hiker travels in 1 hour?*

LISTEN FOR Kyle hiked 6 miles in 2 hours, so divide both by 2 to show he hikes 3 miles in 1 hour. Julio hiked $\frac{1}{2}$ mile in $\frac{1}{4}$ hour, so multiply both by 4 to show he hikes 2 miles in 1 hour.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then have them connect them to the models presented in class.

ASK *How does the solution use unit rates?*

LISTEN FOR For Kyle, the ratio is 6 miles to 2 hours, or 3 miles per hour. For Julio, the ratio is $\frac{1}{2}$ mile to $\frac{1}{4}$ hour, or 2 miles per hour.

For the double number line model, prompt students to identify how the models are labeled to represent the problem.

- *Why do both lines in each model start with zero?*
- *How do you know Kyle travels 3 miles in 1 hour?*

For the fraction model, students should recognize that the unit rates relate miles to hours.

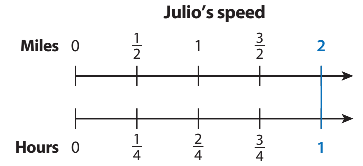
- *Why must you divide miles by hours in each rate?*
- *How can the expressions for unit rates be simplified?*

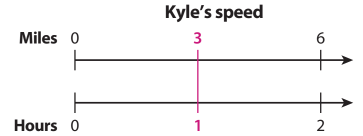
➤ **Explore different ways to understand unit rates for ratios with fractions.**

Julio and Kyle hike along the same trail. Julio hikes the first $\frac{1}{2}$ mile of the trail in $\frac{1}{4}$ hour. Kyle hikes the first 6 miles in 2 hours. Who hikes more quickly?

Model It

You can use a double number line to find each speed.





Model It

You can simplify fractions to find each speed.

Julio's Speed

Julio hikes $\frac{1}{2}$ mile in $\frac{1}{4}$ hour.

The unit rate for the ratio $\frac{1}{2} : \frac{1}{4}$ is $\frac{\frac{1}{2}}{\frac{1}{4}}$.

You can think of $\frac{\frac{1}{2}}{\frac{1}{4}}$ as $\frac{1}{2} \div \frac{1}{4}$.

Kyle's Speed

Kyle hikes 6 miles in 2 hours.

The unit rate for the ratio 6 : 2 is $\frac{6}{2}$.

You can think of $\frac{6}{2}$ as $6 \div 2$.

DIFFERENTIATION | EXTEND

SMP 7

Deepen Understanding
Looking for and Making Use of Structure in Unit Rates

Prompt students to think about the order in which quantities are compared in ratios and unit rates. Ask them to define *reciprocal* and discuss whether it describes $\frac{\frac{1}{2}}{\frac{1}{4}}$ and $\frac{\frac{1}{4}}{\frac{1}{2}}$.

ASK *What is the unit rate for any ratio of Julio's miles to hours? What about the unit rate for any ratio of Julio's hours to miles? What do you notice about these fractions?*

LISTEN FOR $\frac{\frac{1}{2}}{\frac{1}{4}}$ shows the unit rate for any ratio of miles to hours. $\frac{\frac{1}{4}}{\frac{1}{2}}$ shows the unit rate for any ratios of hours to miles. The fractions are reciprocals.

ASK *Suppose you only knew that Julio hiked a miles in b hours. What are the two unit rates you could write for this situation? What quantities do they compare?*

LISTEN FOR You could write $\frac{a}{b}$ for his unit rate in miles per hour and $\frac{b}{a}$ for his unit rate in hours per mile.

Generalize Look for understanding that when you compare two quantities, each unit rate compares the same quantities in a different order. So, you can always write one unit rate as the reciprocal fraction of the other unit rate.

Develop Solving Problems with Unit Rates for Ratios with Two Fractions

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about unit rates.

Before students begin to record and expand on their work in Model It, tell them that problems 3 and 4 will prepare them to provide the explanation asked for in problem 5.

Monitor and Confirm Understanding 1 – 2

- Each double number line relates the distance traveled (shown by the top line) with the time of travel (shown by the bottom line).
- The speed (or rate) of each hiker is shown where 1 hour is marked on the lower number line. Julio's speed is 2 miles per hour, and Kyle's speed is 3 miles per hour.

Facilitate Whole Class Discussion

3 Look for understanding of complex fractions and how they are simplified.

> **ASK** *How can you simplify the complex fraction $\frac{1}{4}$ over $\frac{1}{2}$?*
>
> **LISTEN FOR** Express the fraction as a division problem. Then rewrite it as a multiplication problem: $\frac{1}{4} \times \frac{2}{1}$.

4 Look for understanding of reciprocals in the context of speed.

> **ASK** *How is speed in miles per hour related to speed in hours per mile?*
>
> **LISTEN FOR** The two expressions of speed are reciprocals of each other, such as $\frac{6}{2}$ miles per hour and $\frac{2}{6}$ hour per mile.

5 Look for the idea that ratios can be expressed with either whole numbers or fractions.

> **ASK** *How did your strategy for finding Julio's speed compare with your strategy for finding Kyle's speed?*
>
> **LISTEN FOR** Both strategies are the same, but they use different kinds of numbers.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to find unit rates for ratios with fractions.**

1 Look at the first **Model It**. Who hikes more quickly? How can you use the double number lines to figure out who is hiking more quickly?

Kyle; Possible explanation: Look for the number of miles that corresponds to 1 hour for each boy. The boy who has hiked more miles hikes more quickly.

2 What is Julio's speed in hours per mile? Explain how you can use the double number line to find Julio's speed in hours per mile.

$\frac{1}{2}$ hour per mile; Possible explanation: You can find a ratio for hours per mile. Then you can divide the number of hours by miles to find the rate.

3 Look at the second **Model It**. The fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ describes Julio's speed in miles per hour. What fraction describes Julio's speed in hours per mile? What is his speed in hours per mile?

$\frac{\frac{1}{4}}{\frac{1}{2}}$; $\frac{1}{4} \div \frac{1}{2} = \frac{1}{4} \cdot 2 = \frac{1}{2}$ hour per mile

4 The fraction $\frac{6}{2}$ describes Kyle's speed in miles per hour. What fraction would describe Kyle's speed in hours per mile? What is his speed in hours per mile?

$\frac{2}{6}$; $2 \div 6 = \frac{1}{3}$ hour per mile

5 Do you need a different strategy to find each speed because one involves fractions? Explain.

No; Possible explanation: You can use fractions the same way you use numbers on a double number line or to find a unit rate.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity
Model ratios of fractions.

If students are unsure how to use a ratio with fractions to find a unit rate, then use this activity to connect equivalent ratios, unit rates, and division with fractions.

Materials For each pair: Activity Sheet *Fraction Bars*

- Have students find the unit rate of the ratio $\frac{3}{4} : \frac{1}{8}$ by modeling $\frac{3}{4}$ with one-fourth bars and lining up one-eighth bars below them so that both sets are equal lengths.
- Ask: *How does this model show the ratio $\frac{3}{4} : \frac{1}{8}$?* [The model shows how many times greater $\frac{3}{4}$ is than $\frac{1}{8}$.] Ask: *How does the model show the unit rate?* [The equivalent ratio 6 : 1 shows the unit rate is 6.]
- Ask: *How does the model show that $\frac{3}{4} \div \frac{1}{8}$ is the same as $\frac{3}{4} \times \frac{8}{1}$?* [There are 6 one-eighth sections in $\frac{3}{4}$, and $\frac{3}{4} \times \frac{8}{1} = 6$.]
- Extend by having students practice finding the unit rates of other ratios with fraction bars, such as $\frac{1}{2} : \frac{1}{6}$ and $\frac{6}{8} : \frac{1}{4}$.

Apply It

For all problems, encourage students to use a model to support their thinking. Remind students that models do not need to be perfect to aid in understanding and representing a problem. For example, dividing a number line into precisely equal sections is difficult and not necessary to represent a problem successfully.

7. Students may divide $\frac{1}{3}$ by $\frac{1}{5}$ to find the unit rate.

8. **B is correct.** Students may solve the problem by converting each mixed number to a fraction, and then dividing and simplifying. $\frac{25}{2} \div \frac{15}{2} = \frac{5}{3}$

   A is not correct. This answer is the ratio of the actual area in square meters to the scaled area in square centimeters, instead of the reciprocal ratio.

   C is not correct. This answer is the product of the two areas instead of their ratio.

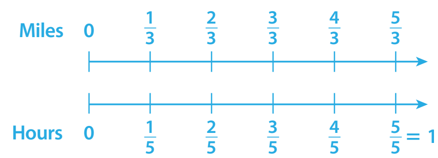
   D is not correct. This answer is the ratio of the units instead of the measurements.

Apply It

➤ **Use what you learned to solve these problems.**

7. A penguin walks south. It completes the first $\frac{1}{3}$ mile in $\frac{1}{5}$ hour and continues walking at the same rate. What is the penguin's speed in miles per hour? Show your work.

   Possible work:



   SOLUTION The penguin's speed is $\frac{5}{3}$ or $1\frac{2}{3}$ miles per hour.

8. A maple tree shades an area of $7\frac{1}{2}$ m². In a scale drawing, the maple tree shades an area of $12\frac{1}{2}$ cm². In the scale drawing, how many square centimeters represent 1 m²?

   A $\frac{3}{5}$

   B $\frac{5}{3}$

   C $93\frac{3}{4}$

   D 10,000

9. Dalila is mixing paste. For every $1\frac{2}{3}$ cups of water, she uses $2\frac{1}{2}$ cups of flour. How much flour does Dalila need for each cup of water? Show your work.

   Possible work:

   $$1\frac{2}{3} \div 2\frac{1}{2} = \frac{5}{3} \cdot \frac{2}{5}$$
   $$= \frac{2}{3}$$

   SOLUTION Dalila needs $\frac{2}{3}$ cup of flour for each cup of water.

CLOSE EXIT TICKET

9. Students' solutions should show an understanding of:
   - expressing the correct quantities in the ratio.
   - converting mixed numbers to fractions.
   - simplifying complex fractions by multiplication.

Error Alert If a student's solution is the reciprocal of the correct answer, then have them reread the problem to identify the amount of flour for each cup of water. Ask them about the ratio of flour to water and what is different about the ratio of water to flour.

Practice Solving Problems with Unit Rates for Ratios with Two Fractions

Problem Notes

Assign **Practice Solving Problems with Unit Rates for Ratios of Two Fractions** as extra practice in class or as homework.

1 **a.** Students should recognize that the units associated with the two fractions show the quantities related by the unit rate. They should identify the unit rate as a ratio of teaspoons of yeast to 1 cup of water, not the reciprocal rate. ***Basic***

b. Students should demonstrate their understanding of unit rates and division. The problem asks students to calculate the number of square feet for each roll, which is equivalent to the rate of square feet per roll. ***Medium***

LESSON 2 | SESSION 2 Name:

Practice Solving Problems with Unit Rates for Ratios with Two Fractions

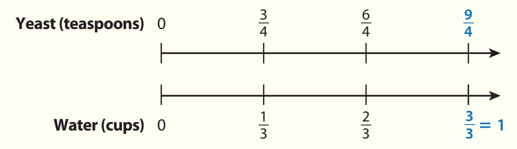
➤ **Study the Example showing how to find the unit rate for a ratio with two fractions. Then solve problems 1–4.**

Example

A baker uses $\frac{3}{4}$ teaspoon of yeast in $\frac{1}{3}$ cup of warm water to make rolls.

How many teaspoons of yeast does the baker use in 1 cup of water?

You can use a double number line to find how many teaspoons of yeast the baker uses in 1 cup of water.



The baker uses $\frac{9}{4}$, or $2\frac{1}{4}$, teaspoons of yeast in 1 cup of water.

1 **a.** In the Example, what is $\frac{\frac{3}{4}}{\frac{1}{3}}$ the unit rate for?

It is the unit rate for how many teaspoons of yeast the baker uses per cup of water.

b. The baker fits 26 rolls onto a pan that measures $1\frac{5}{8}$ ft^2. How many square feet does the baker use for each roll? Show your work. Possible work:

$1\frac{5}{8} = \frac{13}{8}$

There are 26 rolls and $\frac{13}{8}$ ft^2.

$$\frac{\frac{13}{8}}{26} = \frac{13}{8} \div 26$$
$$= \frac{13}{8} \cdot \frac{1}{26}$$
$$= \frac{1}{16}$$

SOLUTION The baker uses $\frac{1}{16}$ ft^2 for each roll.

Vocabulary

equivalent ratios
two ratios that express the same comparison.

unit rate
the numerical part of a rate. For example, the rate 3 miles per hour has a unit rate of 3.

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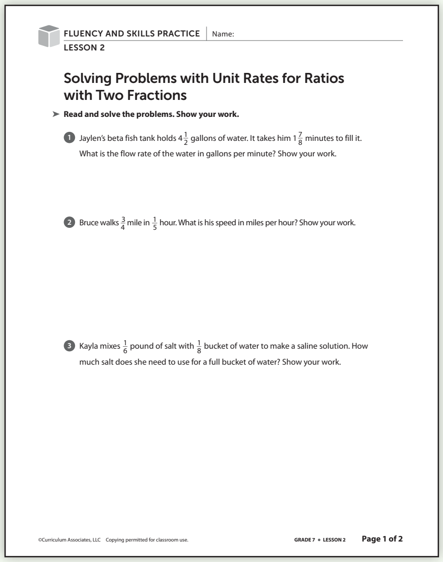
Fluency & Skills Practice

Solving Problems with Unit Rates for Ratios with Two Fractions

In this activity, students solve word problems involving ratios and calculating unit rates. They may use different methods, including drawing double number lines, constructing tables, and simplifying complex fractions.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 2

Solving Problems with Unit Rates for Ratios with Two Fractions

➤ Read and solve the problems. Show your work.

1 Jaylen's beta fish tank holds $4\frac{1}{2}$ gallons of water. It takes him $1\frac{7}{8}$ minutes to fill it. What is the flow rate of the water in gallons per minute? Show your work.

2 Bruce walks $\frac{3}{4}$ mile in $\frac{1}{5}$ hour. What is his speed in miles per hour? Show your work.

3 Kayla mixes $\frac{1}{6}$ pound of salt with $\frac{1}{8}$ bucket of water to make a saline solution. How much salt does she need to use for a full bucket of water? Show your work.

Page 1 of 2

Additional Practice

2 Students may also use the complex fraction $\frac{1}{5}$ divided by $\frac{1}{8}$ to calculate the rate for pounds of clay soil per bale of straw. $\frac{1}{5} \div \frac{1}{8} = \frac{1}{5} \times 8 = \frac{8}{5}$, or $1\frac{3}{5}$ pounds of clay soil per bale of straw. ***Medium***

3 Students may express the mixed number $6\frac{2}{5}$ either as the fraction $\frac{32}{5}$ or as the decimal 6.4. ***Medium***

4 Students should recognize that the solution to this problem involves comparing quantities with a ratio (to calculate the speed, or unit rate, for each animal) as well as comparing quantities by subtraction (to determine how much faster the shark's speed is than the whale's). To find the speeds, students may use a table or bar model divided into tenths (for the shark) or fourths (for the whale). ***Challenge***

LESSON 2 | SESSION 2

2 Kenji mixes $\frac{1}{5}$ pound of clay soil with $\frac{1}{8}$ of a bale of straw to make an adobe brick. How much clay soil will he need to use the whole bale of straw? Show your work. Possible work:


adobe brick

Clay Soil (lb)	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$	$\frac{6}{5}$	$\frac{7}{5}$	$\frac{8}{5} = 1\frac{3}{5}$
Straw (bunch)	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$	$\frac{8}{8} = 1$

SOLUTION He needs $1\frac{3}{5}$ pounds of clay soil.

3 The soccer field at a park is 9,000 square yards. The soccer field takes up $6\frac{2}{5}$ square inches on the map of the park. How many square yards does 1 square inch on the map represent? Show your work. Possible work:

$6\frac{2}{5} = \frac{32}{5}$

$$\frac{9{,}000}{\frac{32}{5}} = 9{,}000 \cdot \frac{5}{32} = \frac{45{,}000}{32} = 1{,}406\frac{1}{4}$$

SOLUTION 1 square inch on the map represents $1{,}406\frac{1}{4}$ square yards.

4 At top cruising speed, a mako shark can swim 4 kilometers in $\frac{1}{10}$ hour. A blue whale can swim 5 kilometers in $\frac{1}{4}$ hour. Which animal swims faster at top cruising speed? How much faster? Show your work. Possible work:

Mako shark: $\frac{4}{\frac{1}{10}} = 4 \div \frac{1}{10} = 4 \cdot 10 = 40$

40 kilometers per hour

Blue whale: $\frac{5}{\frac{1}{4}} = 5 \div \frac{1}{4} = 5 \cdot 4 = 20$

20 kilometers per hour

$40 - 20 = 20$

SOLUTION A mako shark swims 20 kilometers per hour faster than a blue whale at top cruising speed.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** Apply It

MATH TERM

Square feet, or ft^2, is a unit of measure that tells how much space a two-dimensional figure takes up.

ACADEMIC VOCABULARY

To *expand* is to make something larger.

Levels 1–3: Reading/Writing

Help students prepare to write their solutions to Apply It problem 1. Read the problem aloud and clarify the meaning of *expand*. Circle the unit label *ft^2*, *square feet*, and *square foot*. Explain that *foot* is the singular form of *feet*. Then display and read the definition of *square feet*.

Sketch two flower beds and label them *larger* and *smaller*. Point to the sketch as you reread the problem. Help students write using:

- *The larger flower bed is ____ ft^2.*
- *The smaller flower bed is ____ ft^2.*
- *The rate is ____ square feet for every square foot.*

Levels 2–4: Reading/Writing

Help students prepare to write their solutions to Apply It problem 1. Read the problem aloud as students follow along. Point to the unit label *ft^2*. Then have students underline *square feet* and *square foot*. Have students turn and talk about these terms.

Have students make a sketch to help them understand the problem. Have them label the sketch with the words *larger* and *smaller*.

Have students compare the sizes of the flower beds to find the unit rate. Provide this sentence frame:

- *The larger flower bed has ____ for every ____ of the smaller one.*

Levels 3–5: Reading/Writing

Have students read Apply It problem 1 with partners. Have partners prepare for writing their solutions by making a **Co-Constructed Word Bank** to clarify the meaning of any words. Suggest they include the terms *expand, square feet, square foot, smaller,* and *larger,* if needed.

Have partners sketch and label the two flower beds. Have them reread the question and circle the words *larger* and *smaller*. Have students solve the problem and use **Stronger and Clearer Each Time** to explain their solutions.